Formal Proofs:

These notes contain the five formal proofs you must know, understand and learn off by heart for the Junior Certificate Examination. There is a very high chance at least one of these proofs will be asked in the exam so it is imperative you have learnt them off as they are easy marks.

There is no easy way about to do this other than just memorize all five. Test yourself continuously throughout the year on this five and make sure this is just an ease by the end.

In a proof you are writing logical steps to prove a theorem. Each step must be in the write order.

These are the five:

- To prove the angles in any triangle add up to 180°
- To prove the measure of the angle at the centre of the circle is the measure of the angle at the circumference standing on the same arc.
- To prove in a parallelogram, opposite sides and angles are equal
- To prove in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two sides
- To prove an exterior angle of a triangle equals the sum of two opposite angles
The angles in any triangle add up to \( 180^\circ \).

**Diagram:**

**Given:** Triangle ABC with angles 1,2 and 3.

**To prove:** \( (\angle b) + (\angle c) + (\angle a) = 180^\circ \)

**Construction:** Draw a line through A parallel to BC, label angles d and e.

**Proof:**

\( (\angle b) = (\angle d) \) and \( (\angle c) = (\angle e) \) (Alternate angles).

\( (\angle d) + (\angle a) + (\angle e) = 180^\circ \) (Straight angle).

\( (\angle b) + (\angle a) + (\angle c) = 180^\circ \) (Since \( (\angle d) = (\angle b) \) and \( (\angle e) = (\angle c) \)).

\[ \therefore (\angle b) + (\angle c) + (\angle a) = 180^\circ \]
The measure of the angle at the centre of the circle is the measure of the angle at the circumference standing on the same arc.

Diagram:

Given: A circle with centre O, containing points A, B & C

To prove: \( \angle BOC = 2 \angle BAC \)

Construction: Join A to O and continue to D. Label angles 1, 2, 3, 4 & 5.

Proof: Consider Triangle AOB

\( \angle 1 = \angle 2 + \angle 3 \) (Exterior angle)
\( \angle 2 = \angle 3 \) (Equal radii – isosceles triangle)
\( \angle 1 = \angle 2 + \angle 2 \)
\( \angle 1 = 2 \angle 2 \)

Similarly \( \angle 4 = 2 \angle 5 \)

\( \angle 1 + \angle 4 = 2(\angle 2) + 2(\angle 5) \)
\( \angle 1 + \angle 4 = 2(\angle 2 + \angle 5) \)
\( \therefore \angle BOC = 2 \angle BAC \)
In a parallelogram, opposite sides and angles are equal

Diagram:

**Given:** A parallelogram ABCD

**To prove:** $|AB| = |DC|, |AD| = |BC|, (\angle ADC) = (\angle ABC), (\angle DCB) = (\angle DAB)$

**Construction:** Join A to C, label angles 1,2,3,4.

**Proof:** Consider triangle ABC and triangle ADC

$(\angle 1) = (\angle 3)$ and $(\angle 2) = (\angle 4)$ *(Alternate angles)*

$|AC| = |AC|$ *(Common side)*

\[\therefore\] Triangle ABC is congruent to $(\equiv)$ triangle ADC *(ASA – Angle, side, angle).*

\[\therefore\] $|AB| = |DC|$ and $|AD| = |BC|$ *(Corresponding sides)*, and $(\angle ADC) = (\angle ABC)$ *(Corresponding sides)*. Similarly, $(\angle BAD) = (\angle BCD)$
In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two sides.

Diagram:

**Given:** A right angled triangle with \( \angle ABC = 90^\circ \)

**To prove:** \((AC)^2 = (AB)^2 + (BC)^2\)

**Construction:** Draw \( BD \perp (\text{Perpendicular to}) \ AC \)

**Proof:** Consider the triangles \( ABC \) and \( ABD \).

\( \angle ABC = \angle BDA \) (Both 90\(^\circ\))

\( \angle BAC = \angle BAD \) (Common angle)

\( \angle BCA = \angle ABD \) (Remaining angle)

\( \therefore \) Triangle \( ABC \) & Triangle \( ABD \) are similar.

\( AB = AC \)
Formal Proofs

(AD) = (AB)

(AB) x (AB) = (AC) x (AD)

(AB)² = (AC) x (AD)

Consider the triangles ABC and BCD:

(∠ABC) = (∠BDC) (Both 90°)
(∠BCA) = (∠BCD) (Common angle)
(∠BAC) = (∠DBC) (Remaining angle)

∴ Triangle ABC & Triangle BCD are similar

(BC) = (AC)
(DC) = (BC)

(BC) x (BC) = (AC) x (DC)

(BC)² = (AC) x (DC)

(AB)² + (BC)² = (AC) x (AD) + (AC) x (DC)

(AB)² + (BC)² = (AC) x ((AD) + (DC))

(AB)² + (BC)² = (AC) x (AD) x (AC) (Since (AD) + (DC) = (AC))

(AB)² + (BC)² = (AC)²

(AC)² = (AB)² + (BC)²
An exterior angle of a triangle equals the sum of two opposite angles

Diagram:

Given: Triangle ABC with interior angles 1, 2 and an exterior angle 4.

To prove: \( \angle 1 + \angle 3 = \angle 4 \)

Proof: \( \angle 2 + \angle 4 = 180^\circ \) (Straight angle).

\( \angle 1 + \angle 2 + \angle 3 = 180^\circ \) (Angles in a triangle add up to 180°)

\( \angle 1 + \angle 2 + \angle 3 = \angle 2 + \angle 4 \) (Both 180°)

\( \angle 1 + \angle 3 = \angle 4 \) (By cancelling \( \angle 2 \) on both sides)